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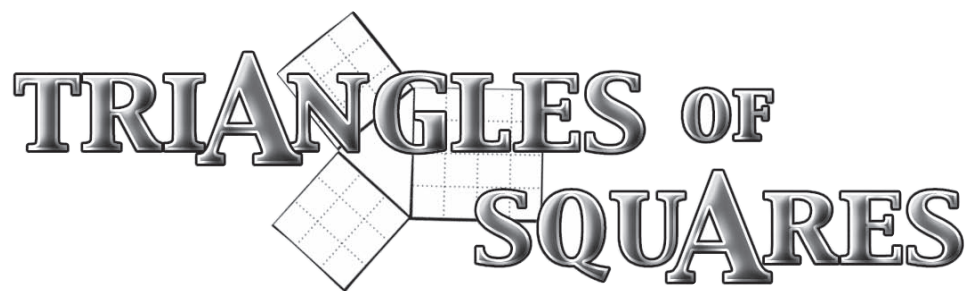
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TRIANGLES OF SQUARES



Topic

Algebra, Geometry—Pythagorean Theorem

Key Questions

1. If you know the lengths of the three sides of a triangle, how can you determine if the triangle is equilateral, isosceles, or scalene?
2. If you know the lengths of the three sides of a triangle, how can you determine if the triangle is acute, right, or obtuse?

Learning Goals

Students will:

- learn to sort triangles by angles and sides, and
- recognize the equality and inequalities between the sum of the areas of the squares on the short legs of a triangle compared to the area of the square on the longest side

Guiding Documents

Project 2061 Benchmarks

- *Some shapes have special properties: Triangular shapes tend to make structures rigid, and round shapes give the least possible boundary for a given amount of interior area. Shapes can match exactly or have the same shape in different sizes.*
- *Mathematicians often represent things with abstract ideas, such as numbers or perfectly straight lines, and then work with those ideas alone. The “things” from which they abstract can be ideas themselves (for example, a proposition about “all equal-sided triangles” or “all odd numbers”).*
- *Graphs can show a variety of possible relationships between two variables. As one variable increases uniformly, the other may do one of the following: always keep the same proportion to the first, increase or decrease steadily, increase or decrease faster and faster, get closer and closer to some limiting value, reach some intermediate maximum or minimum, alternately increase and decrease indefinitely, increase and decrease in steps, or do something different from any of these.*

*NCTM Standards 2000**

- *Understand relationships among the angles, side lengths, perimeters, areas, and volumes of similar objects*

- *Use geometric models to represent and explain numerical and algebraic relationships*
- *Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships*
- *Use graphs to analyze the nature of changes in quantities in linear relationships*
- *Create and critique inductive and deductive arguments concerning geometric ideas and relationships, such as congruence, similarity, and the Pythagorean relationship*

Math

Geometry

categories of triangles
Pythagorean Theorem

Algebra

graphing
equalities/inequalities

Integrated Processes

Observing
Comparing and contrasting
Generalizing

Materials

Option One:

cardstock
scissors

Option Two:

AIMS Algebra Tiles
2 cm squares

Background Information

Finding all the possible triangular regions that can be surrounded from a limited variety of discrete size squares can provide a rich understanding of triangles.

Initially students will discover that not all combinations of three squares work. In order to form a triangle, the combined edges of two shorter squares must be greater than the length of the edge of the largest square. With the four different size squares of 2, 3, 4, and 5 centimeters, 20 different combinations can be made. Three of these, however, will not form triangles (2,2,4; 2,2,5; 2,3,5).

Constructing 17 different triangles provides an excellent opportunity to reinforce the understanding of types of triangles. When sorting by sides, students will

find four equilateral, three scalene, and ten isosceles triangles. When sorting by angles, students will be able to find one right, four obtuse, and 12 acute triangles.

Having students consider the sum of the areas of squares made on the two shorter legs of a triangle ($a^2 + b^2$) compared to the area of the square formed on the longest leg (c^2) develops a full concept of the Pythagorean Theorem ($a^2 + b^2 = c^2$). Placing these measurements alongside the types of triangles by angles quickly shows the following:

- when $a^2 + b^2 > c^2$, it is an acute triangle,
- when $a^2 + b^2 = c^2$, it is a right triangle,
- when $a^2 + b^2 < c^2$, it is an obtuse triangle.

Discovering these equalities and inequalities helps students understand that the Pythagorean Theorem is true only for right triangles. It also provides a way to interpret the broader relationship of ($a^2 + b^2$) to (c^2).

By graphing the measurements, the relationship is reinforced. As students plot each triangle's point with ($a^2 + b^2$) being the vertical axis and (c^2) being the horizontal axis, they will discover that only the right triangle falls on the line with a slope of one. The acute triangles all plot above the one line and the obtuse triangles below the line. This accentuates the special relationship of the right triangle while providing a very pertinent use of graphing inequalities.

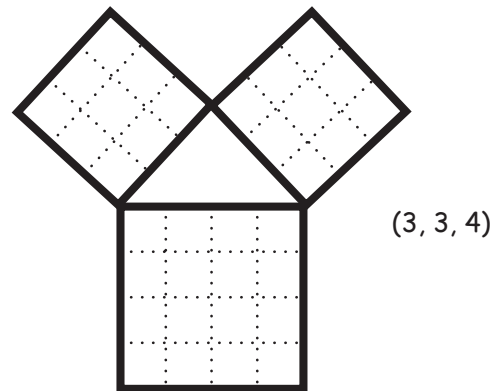
Management

- Before the lesson, determine what materials will be used for squares. *Option One:* The blackline master may be copied onto cardstock and students may cut out the squares when they are needed. *Option Two:* AIMS Algebra Tiles, which provide a more kinesthetic appeal for students, may be used; however, the teacher will need to cut 2 cm square pieces before beginning the lesson.
- Students should be familiar with the types of triangles by sides (equilateral, isosceles, scalene) and angle (acute, right, obtuse) before beginning this investigation. If they are not, some instruction should be done to familiarize them with all the types.
- Emphasize to the students that they need to be careful when forming the triangles. The corners of the squares must touch each other to form a vertex. Several of the acute and obtuse triangles will be mistakenly classified as right triangles if they are not constructed accurately. A corner from a piece of paper will help students check right angles.
- Partners work very well for this activity. One student can construct while the second records.

Procedure

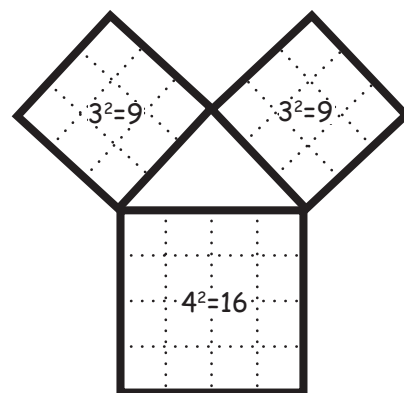
- Distribute the materials and allow students time to cut out the squares if necessary. If the materials are not labeled for length and area, have the students examine and compare the pieces to recognize the sizes are 2, 3, 4, and 5 centimeters.

- Using the squares have students determine how many different-sized triangles can be made from these four sizes of squares.



Although many students will begin by randomly developing combinations, encourage them to develop an ordered listing to assure they have found all the possible combinations. They should discover that three combinations of squares do not form triangles (2,2,4; 2,2,5; 2,3,5).

- When students have confirmed that a combination of squares forms a triangle, direct them to record the lengths on the chart.
- Encourage the students to carefully form the triangles so the corners of the squares touch exactly. Have them determine and record the type of triangle that is formed. (A corner of a paper can be used to determine right angles.)
- Optional: Have students cut squares of paper and glue them to make permanent records of all possible triangles.
- Direct the students to calculate the area of the square on each edge of the triangle. Have them record the sum of the areas of the squares of the two shorter legs in one column of the chart and the area of the square on the longest leg in another column of the chart.



- Tell them to choose the equality/inequality sign that makes the number sentence true.
- Hold a discussion in which students share observations they make about the relationships of number sentences and types of triangles.

9. Have them make a graph of the data by plotting the areas of the squares on the sides of a triangle.
 10. Have them observe and record the relationship of the position where each triangle is represented on the graph and the type of triangle it is. Direct them to record the algebraic equation that represents the line that is drawn on the graph [$a^2 + b^2 = c^2$ or $= 1$] and inequalities describing the regions above [$a^2 + b^2 > c^2$ or > 1] and below [$a^2 + b^2 < c^2$ or < 1] the line.
4. What is the relationship of the position of a triangle's point on the graph and the type of triangle it is by angle? [right on the one line, acute above the one line, obtuse below the one line]
 5. If you know the lengths of the edges of a triangle, how can you determine what type of a triangle it is by angles? [Find the squares on all three sides. If the sum of the squares on the shortest two sides equals the square on the longest side, it is a right triangle. If the sum is greater, it is an acute triangle. If the sum is less, it is an obtuse triangle.]
 6. What are you wondering now?

Connecting Learning

1. How can you tell if a combination of squares will make a triangle before assembling them? [To make a triangle, the combination of the lengths of the two shorter sides must be greater than the length of the longest side ($a + b > c$).]
2. How can you look at the lengths of the sides of a triangle and determine what type of triangle it is by sides?
3. What relationship do you see in squares on the sides of a triangle and the type of triangle it is by angles? [$a^2 + b^2 > c^2$: acute; $a^2 + b^2 = c^2$: right; $a^2 + b^2 < c^2$: obtuse]

Extension

Measure triangles found in the classroom and use the algebraic sentences to determine their types. Confirm that it is correct.

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TRIANGLES OF SQUARES

Key Questions

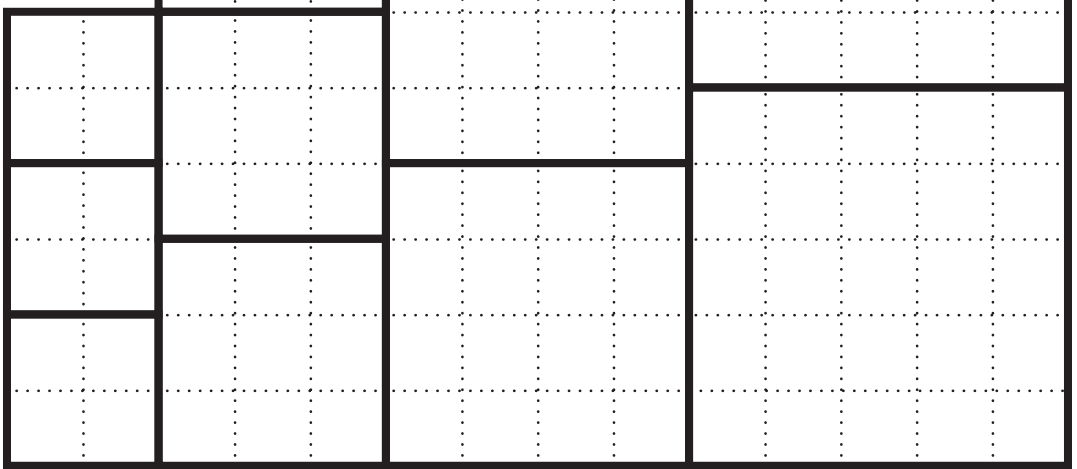
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Learning Goals

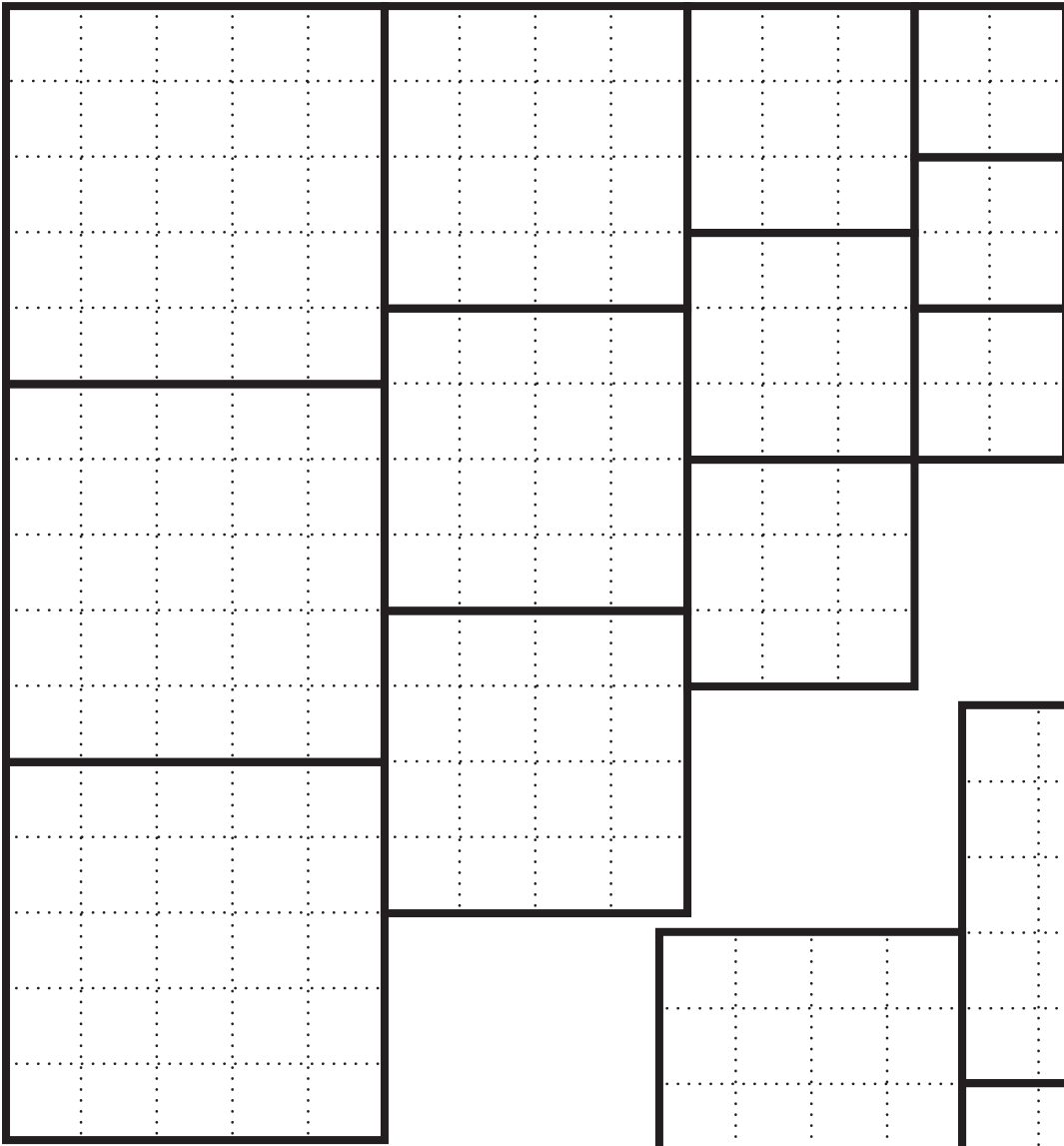
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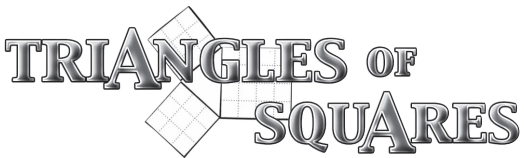
Triangles
of
Squares



Triangles
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Squares



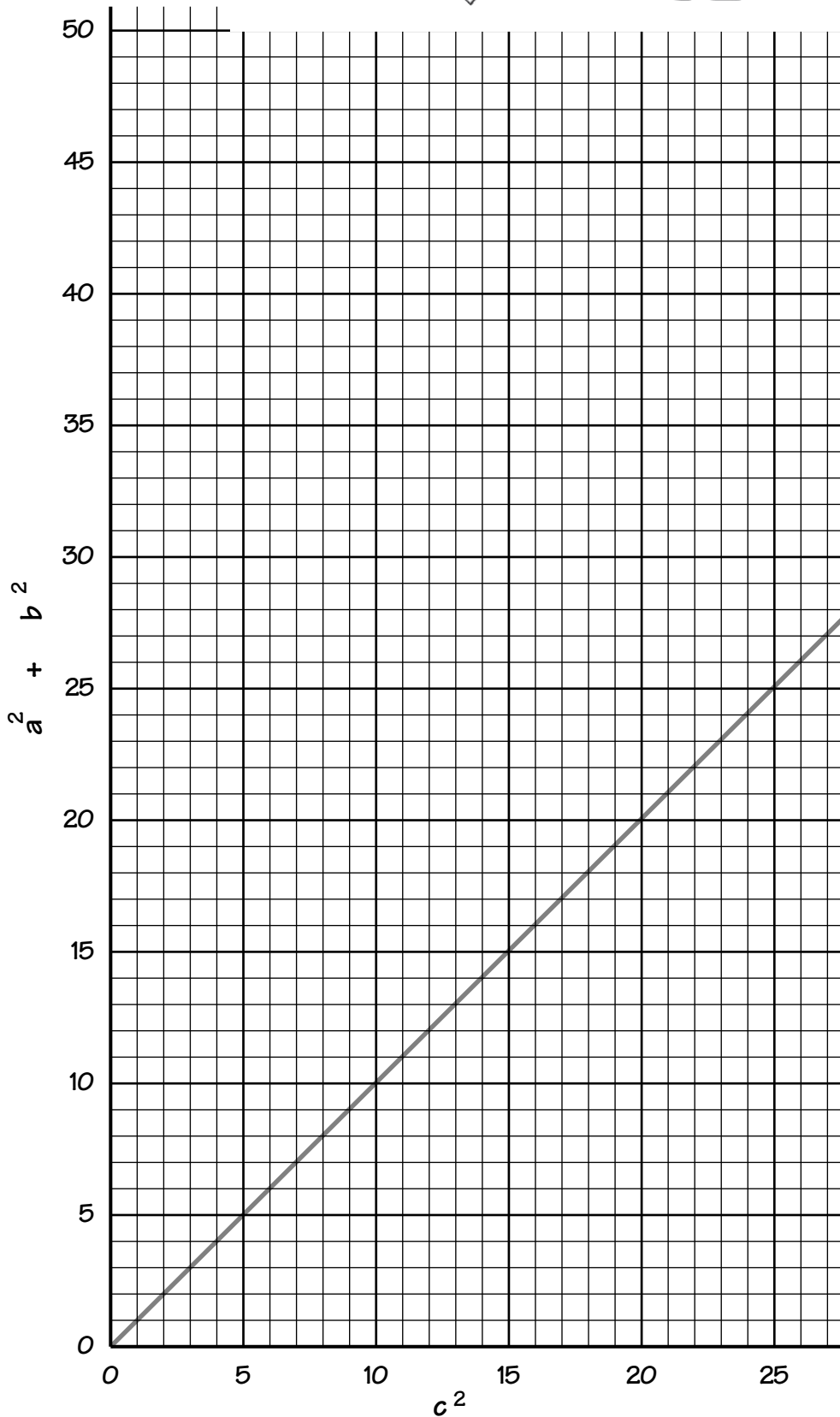
TRIANGLES OF SQUARES



Make all the possible different size triangles you can out of 2,3,4, and 5 cm squares. Record the dimensions and complete the chart.

Length of Sides			Squares on Sides			Type of Triangle	
Shortest Side a	Mid-length Side b	Longest Side c	Sum of squares on shorter sides a^2+b^2	< > =	Area of square on longest side c^2	By Angles Acute Right Obtuse	By Sides Equilateral Isosceles Scalene

TRIANGLES OF SQUARES

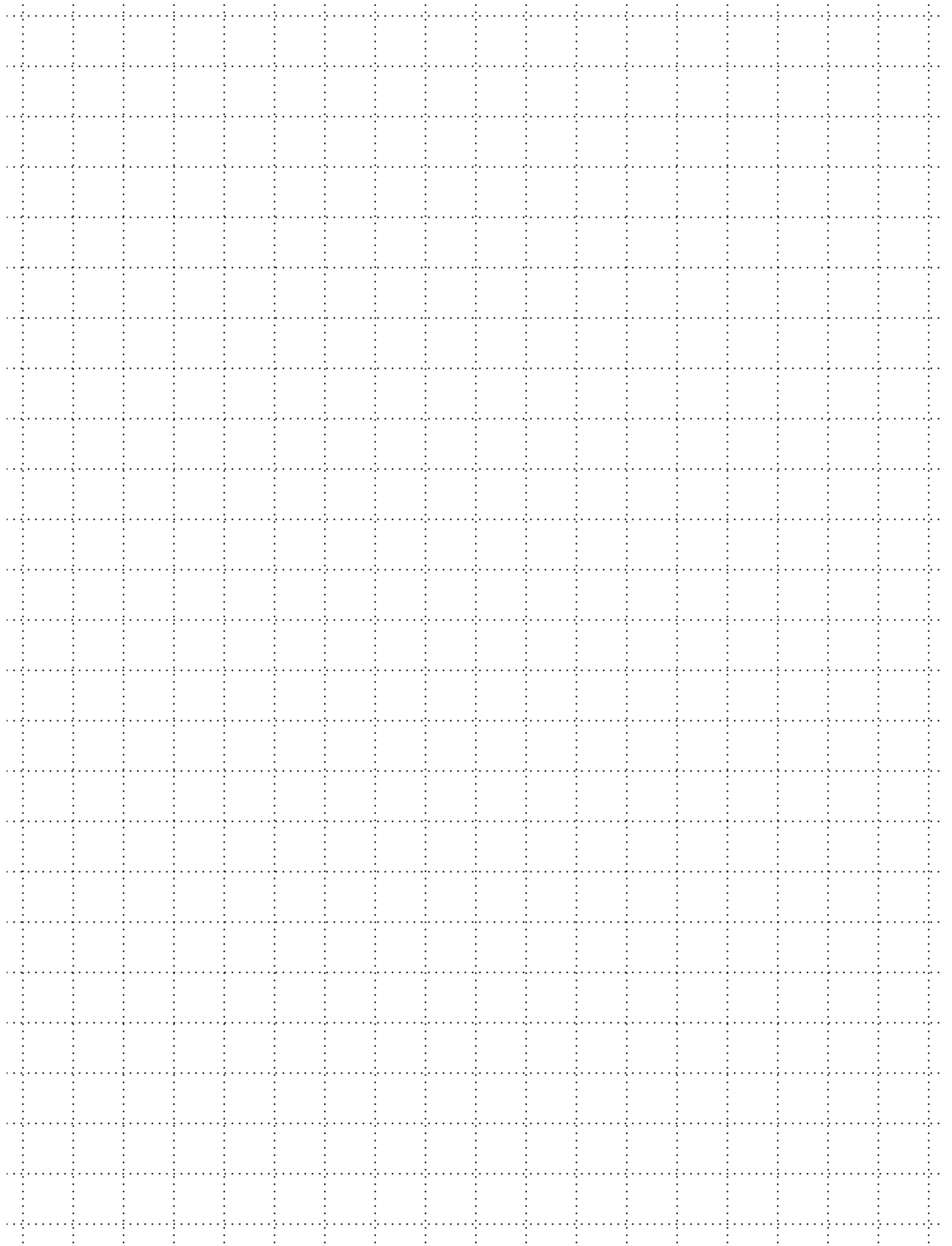


Plot a point for each triangle so it records the area of sum of the squares on the two smaller sides and the area of the square on the longest side.

a. What type of triangles get plotted above the gray line?

b. What type of triangles get plotted on the gray line?

c. What type of triangles get plotted below the gray line?



TRIANGLES OF SQUARES

Connecting Learning

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