Department of Education

Multiplying and Dividing in Scientific Notation – Grade Eight

Ohio Standards Connection:

Number, Number Sense and Operations

Benchmark I
Estimate, compute and solve problems involving scientific notation, square roots and numbers with integer exponents.

Indicator 8
Add, subtract, multiply, divide and compare numbers written in scientific notation.

Related Benchmark A
Use scientific notation to express large numbers and numbers less than one.

Related Indicator 1
Use scientific notation to express large numbers and small numbers between 0 and 1.

Mathematical Processes Benchmarks

E. Use a variety of mathematical representations flexibly and appropriately to organize, record and communicate mathematical ideas.
F. Use precise mathematical language and notation to represent problem situations and mathematical ideas.
G. Write clearly and coherently about mathematical thinking and ideas.

***Lesson Summary**:*
In this lesson, students make conjectures to discover how to multiply and divide numbers written in scientific notation. Students perform operations on numbers written in scientific notation or convert numbers to scientific notation and perform operations. Students apply scientific notation to measurement concepts. Flexible grouping options, cooperative learning strategies and multiple representations are embedded throughout the lesson to address auditory, kinesthetic and visual learning preferences.

***Estimated Duration**: One hour 30 minutes to two hours*

Commentary:
Students make conjectures for multiplying and dividing numbers in scientific notation by analyzing representative whole number operations and patterns. Asking students to make conjectures based on their analyses places a greater demand on their thinking process. This demand results in students being able to retrieve and retain the procedures of these operations. Using real-world contexts such as population and economic data, science investigation and measurement enables deeper understanding and relevancy of the concept.

Pre-Assessment:
This pre-assessment determines if students can use scientific notation to express large numbers and positive numbers less than one. Students demonstrating a lack of familiarity, exposure or understanding of these topics may require scaffolding to ensure success with the content presented in this lesson.

- Distribute Attachment A, *What I Know about Scientific Notation*, and have students complete the exercises. Allow calculators as a resource for students who demonstrate difficulty writing numbers in scientific notation.
- Monitor students as they complete the exercises, making notes of common errors and students who do not demonstrate the ability to write numbers in scientific notation. Assist students and ask clarifying questions as needed.

- Check the pre-assessment exercises as a class and ask questions that target common errors observed. Ask students to give a non-verbal signal (thumbs-up/thumbs-down) to show whether they agree or disagree with student responses.

Scoring Guidelines:

Option One: Informal Observation
Informally evaluate students' strengths and weaknesses while they complete the pre-assessment. If students struggle with writing numbers in scientific notation, provide appropriate intervention.

Option Two: Scoring Rubric
Pre-assessments scored as a zero, one or two indicate intervention is necessary for students. Provide sufficient intervention as necessary.

4 points	All four numbers are correctly rewritten using scientific notation. Explanation is accurate and clearly communicated.
3 points	Three of four numbers are correctly rewritten using scientific notation. Explanation, while clearly communicated, reflects one minor error in understanding.
2 points	One of two or the numbers are correctly rewritten using scientific notation. Explanation contains multiple minor errors which indicate minor gaps in understanding.
1 point	No numbers are rewritten correctly using scientific notation. Explanation contains major errors which translate into major gaps in understanding.
0 points	No evidence of understanding is demonstrated through the pre-assessment. No attempt is made to complete the pre-assessment.

Answer Key:

1. 2×10^3
2. 3.124×10^5
3. 2×10^{-2}
4. 4.15×10^{-4}

Post-Assessment:

- Distribute Attachment B, *Multiplying and Dividing Numbers in Scientific Notation* to each student. Have students complete the exercises.

Scoring Guidelines:

A rubric for scoring the students' work and an answer key are found on Attachment C, *Post-Assessment Scoring Guidelines and Answer Key*.

Instructional Procedures:

Part One

1. Complete the pre-assessment activity.
2. Distribute *Multiplying Numbers Written in Scientific Notation*, Attachment D. Assign Exercises one through six. Model exercises to clarify procedures and make conjectures about the connection between whole numbers and numbers written in scientific notation.

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Instructional Tip:
This allows students to generate conjectures about how to multiply numbers written in scientific notation, based on the patterns found when completing the exercises.

3. Circulate while students work in pairs and provide support to pairs having difficulty. Make anecdotal or mental notes about conjectures students make, common errors and students who demonstrate difficulty with numbers in scientific notation.
4. Facilitate a class discussion to share the correct answers and make a conjecture about multiplying numbers written in scientific notation. At this point, formally introduce the Product of Powers rule: $10^a \times 10^b = 10^{a+b}$. Provide students with more exercises of multiplying numbers in scientific notation if additional instruction is needed.
5. Direct pairs of students to complete exercises seven to 11 on Attachment D, *Multiplying Numbers Written in Scientific Notation.* Observe students and provide individual support to pairs having difficulty. Observe students making accurate conjectures and call on those students during the whole-class discussion. Facilitate a whole-class discussion. Elicit the correct answers along with the correct conjectures from the students.
6. Have students use scientific notation to find the area of their desktop and the classroom.
   a. Review the formula for finding area.
   b. Ask students which metric unit of measurement would be reasonable for measuring the desk. (centimeters)
   c. Tell students to use measurement tools to find the length and width of their desktop and calculate the area. Record the measurements in scientific notation.
   d. Direct students to discuss with a partner the following questions:
      - What is the area of the desktop using millimeters as the measurement unit? Kilometers?
      - How can scientific notation be used to solve the problems?
   e. Ask students to share their responses to the questions and summarize a procedure for students to convert measurement units, using scientific notation. Students record procedures and strategies in journals or notebooks.

Instructional Tip:
Students should recognize that:
- one meter is equal to 10^2 centimeters
- one centimeter is equal to 10^{-2} meters
- one meter is equal to 10^{-3} kilometers
- 1 kilometer is equal to 10^3 meters

Part Two
7. Distribute *Dividing Numbers in Scientific Notation to Students*, Attachment E, to students. Assign pairs of students Exercises one through six. Model the first exercise to clarify procedures and stress the connection between whole numbers and numbers written in scientific notation.

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Instructional Tip:
The purpose of this activity is for students to generate conjectures about how to divide numbers written in scientific notation, based on the patterns found when completing the exercises.

8. Observe and provide support to students having difficulty. Make anecdotal or mental notes about conjectures students make, common errors and students who demonstrate difficulty with numbers in scientific notation.
9. Facilitate a whole class discussion. Elicit the correct answers along with the correct conjectures from the students. Summarize conjectures. At this point, formally introduce the Quotient of Powers rule: $10^a \div 10^b = 10^{a-b}$. Provide additional exercises if needed.
10. Direct students to complete questions seven through 11. Circulate while students work in pairs and provide individual support to pairs having difficulty. Observe students making accurate conjectures and call on those students during the whole-class discussion. Facilitate a whole-class discussion. Elicit the correct answers along with the correct conjectures from the students.
11. Summarize *Multiplying and Dividing Numbers in Scientific Notation*, Attachment F. Instruct students to complete a graphic organizer which compares multiplying and dividing numbers in scientific notation. If students are not familiar with using graphic organizers in mathematics, model the use and discuss the structure of the organizer. Allow small groups to discuss concepts or ideas that may be included in the organizer. Individuals should complete the organizer, using their own words and mathematical language.

Instructional Tip:
Have students who exceed the indicator's expectations create an original graphic organizer or visual representation (poster). There are software programs available that create graphic organizers. Using computer software, students can create their graphic organizers using an appropriate program.

12. Distribute *National Debt*, Attachment H. Depending on time, students can complete the exercises in class or at home. An answer key is provided on *National Debt Answer Key*, Attachment I.
    a. Students should complete the problems working in pairs.
    b. Select students to present their correct answers and work to the class.
    c. Have students share problems they created with another pair.
    d. Provide time for students to solve problems and check their work.

Instructional Tip:
Using the computer, have students research the national debt and the population of the United States to update numbers for population and debt. For the problem students need to create, direct them to research another related fact and use it to develop their own problem.

13. Have students complete a "quick-write" summarizing multiplication and division in scientific notation. Complete explanations include conjectures made during the lesson and address correction for exponents.

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Differentiated Instructional Support:
Instruction is differentiated according to learner needs, to help all learners either meet the intent of the specified indicator(s) or, if the indicator is already met, to advance beyond the specified indicator(s).

- Differentiate for students still developing understanding of the content and those that demonstrate advanced understanding by the size of numbers used and by the number of problems learners complete.
- Students that experience success early in the lesson can research situations when scientific notation would be helpful in calculations and when it would not make sense to use it. Once this evaluation has taken place, students can research specific examples such as space explorations and exponential growth or decay to back up their evaluation.
- Present students experiencing difficulties with concepts with adjusted assignments to meet their needs. Adaptations for this lesson could include a partially completed graphic organizer to be filled in or a partially filled-in conjecture page for multiplying and dividing.
- Cooperative learning strategies make all students accountable and allow auditory and kinesthetic learners an avenue for reaching the goals of the lesson.
- Present students in need of intense intervention models of numbers using base-ten blocks or visual representations of base-ten blocks. Begin by having students represent whole numbers with the models or visual representations. Relate the size of the blocks to the exponent; a cube (1 unit), representing the zero power, a rod (10units), the first power, a flat (100 units), the second power and a large cube (1000 units), the third power. Scaffold understanding by having them represent numbers in scientific notation. Although the blocks have limitations, they provide access to understanding the basis of the concept for a variety of learning preferences.

Extensions:

- Students examine additional powers of 10 and their prefixes such as atto-, nano-, and giga-.
- Students research both the national debt and world population. Students examine the growth of each and display their conclusions graphically which could lead into a discussion about exponential growth.

Home Connections and Homework Options:

- *National Debt*, Attachment H
- *Area in My Home*, Attachment J
- Research a topic of the student's choice on the Internet and create questions involving multiplying and dividing numbers in scientific notation.

Interdisciplinary Connections:

- **Science**: Students use scientific notation to express mass, volume, energy, distance, charge, area and half-life.
- **Economics**: Students examine the growth of the National Debt graphically.

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Materials and Resources:
The inclusion of a specific resource in any lesson formulated by the Ohio Department of Education should not be interpreted as an endorsement of that particular resource, or any of its contents, by the Ohio Department of Education. The Ohio Department of Education does not endorse any particular resource. The Web addresses listed are for a given site's main page, therefore, it may be necessary to search within that site to find the specific information required for a given lesson. Please note that information published on the Internet changes over time, therefore the links provided may no longer contain the specific information related to a given lesson. Teachers are advised to preview all sites before using them with students.

For the teacher: Measurement tools

For the student: Calculators, meter sticks, rulers, trundle wheels, tape measures

Vocabulary:
- mantissa
- power
- scientific notation

Technology Connections:
- Use Web sites which collect large numbers of statistical data relevant to eighth-graders, such as highest grossing movies and albums, attitudinal surveys, current events, etc.
- Use calculators to verify students' calculations for exercises involving multiplication and division of numbers in scientific notation.

Research Connections:
Burke, Jim. *Tools for Thought*: *Graphic Organizers for Your Classroom*. Portsmouth, N.H.: Heinemann, 2002.

Marzano, Robert J., Jane E. Pollock and Debra Pickering. *Classroom Instruction that Works: Research-Based Strategies for Increasing Student Achievement*, Alexandria, Va: Association for Supervision and Curriculum Development, 2001.

Attachments:
Attachment A, *What I Know About Scientific Notation*
Attachment B, *Multiplying and Dividing Numbers in Scientific Notation Post-Assessment*
Attachment C, *Post-Assessment Scoring Guidelines and Answer Key*
Attachment D, *Multiplying Numbers Written in Scientific Notation*
Attachment E, *Dividing Numbers Written in Scientific Notation*
Attachment F, *Multiplying and Dividing Numbers in Scientific Notation*
Attachment G*, Answer Key for Attachments D, E and F*
Attachment H, *The National Debt*
Attachment I, *National Debt Answer Key*
Attachment J, *Area in My Home*

Multiplying and Dividing in Scientific Notation – Grade Eight

Attachment A
What I Know About Scientific Notation

Name_____ **Date**_____

Directions:

- For each problem below, rewrite the given number using scientific notation.
- Explain your thought process for each solution.

1. 2000 = _____
   Explain your thinking:

2. 312,400 = _____
   Explain your thinking:

3. 0.02 = _____
   Explain your thinking:

4. 0.000415 = _____
   Explain your thinking:

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Attachment B
Multiplying and Dividing Numbers in Scientific Notation Post-Assessment

Name_____ **Date**_____

Directions: Multiply or divide the following and write your answers in scientific notation. Show all of your work or explain your steps.

1. $(7.7 \times 10^{3})\ (8.4 \times 10^{4}) =$

2. $(4.8 \times 10^{-5})\ (5.5 \times 10^{-6}) =$

3. $\dfrac{3.4 \times 10^{5}}{1.7 \times 10^{3}} =$

4. $\dfrac{4.0 \times 10^{-6}}{8.0 \times 10^{-3}} =$

Directions: Find the area of the rectangular region that has the given dimensions. Show all of your conversions, set up and solution for each problem. Write your answers in scientific notation.

5. Length = 10.9 m
   Width = 4.9 m
   Area in km^2 =

6. Length = 8.3 m
   Width = 4.7 m
   Area in cm^2 =

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Attachment C
Post-Assessment Scoring Guidelines and Answer Key

Post-Assessment Rubric

	3	2	1	0
Approach	Approach to solution reflects thorough understanding of how to solve the problem.	Approach reflects some understanding of how to solve the problem.	Approach reflects minimal understanding of how to solve the problem.	Approach reflects no understanding of how to solve the problem.
Computations	Accurate computations lead to an accurate answer.	Computations include one minor error.	Computations include multiple minor errors.	Computations include one or more major errors.
Reasonableness	Answer is reasonable. Well thought out explanation or organized work accompanies solution.	Answer is reasonable. Explanation or work accompanies solution.	Answer is somewhat reasonable. Explanation or work lead to given answer.	Answer is not reasonable. Explanation or work is unclear or confusing.

Answer Key:

1. 6.468×10^{8}
2. 2.64×10^{-10}
3. 2.0×10^{8}
4. 5.0×10^{-10}
5. 5.341×10^{-5}
6. 3.901×10^{5}

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Attachment D
Multiplying Numbers Written in Scientific Notation

Name_____ **Date**_____

Directions: For each exercise below, complete the multiplication in the first row and write the answer in the box to the right of the equals sign. In the second row, covert each whole number into scientific notation and write it in the box below. Look for patterns in the answers and make conjectures about multiplying numbers in scientific notation.

1. 20 × 300 = ☐

☐ × ☐ = 6×10^3

2. 25 × 100 = ☐

☐ × ☐ = 2.5×10^3

3. 0.01 × 0.003 = ☐

☐ × ☐ = 3×10^{-5}

4. 0.2 × 0.0024 = ☐

☐ × ☐ = 4.8×10^{-4}

5. Use your work and solutions in exercises 1 - 4 to make a conjecture about multiplying numbers written in scientific notation.

Attachment D (Continued)
Multiplying Numbers Written in Scientific Notation

6. Make your own problem involving multiplying numbers written in scientific notation and see if your directions work. If they don't work, revise them.

Note: Sometimes when multiplying numbers in scientific notation, there must be a correction for exponents. The following two problems involve correcting for exponents. Fill in the boxes as in numbers 1 - 4 and see if you can figure out what correcting for exponents means.

7. $90 \times 200 = \square$

$\downarrow \qquad \downarrow$

$\square \times \square = 1.8 \times 10^4$

8. $5000000 \times 300000 = \square$

$\downarrow \qquad \downarrow$

$\square \times \square = 1.5 \times 10^{12}$

9. How are questions seven and eight different from questions 1 - 4?

10. When do you need to worry about correcting for exponents?

11. Based on your answer to question 10, create a problem that you need to correct for exponents.

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Attachment E
Dividing Numbers Written in Scientific Notation

Name_____ **Date**_____

Directions: For each exercise below, complete the division problem with whole numbers in the first row and write the answer in the box to the right of the equal sign. In the second row, covert each whole number into scientific notation and write it in the box below. Look for patterns in the answers and make conjectures about dividing numbers in scientific notation.

1. 400 ÷ 25 = ☐

☐ ÷ ☐ $= 1.6 \times 10^{1}$

2. 3200 ÷ 20 = ☐

☐ ÷ ☐ $= 1.6 \times 10^{2}$

3. 3000000 ÷ 15000 = ☐

☐ ÷ ☐ $= 2 \times 10^{2}$

4. 30000 ÷ 1500000 = ☐

☐ ÷ ☐ $= 2 \times 10^{-2}$

5. Use your findings in questions 1 - 4 to make a conjecture about dividing numbers written in scientific notation.

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Attachment E (Continued)
Dividing Numbers Written in Scientific Notation

6. Make your own problem involving dividing numbers written in scientific notation and see if your directions work. If they don't work, revise them.

Note: Sometimes when dividing numbers in scientific notation there needs to be a correction for exponents. The following two problems involve correcting for exponents. Fill in the boxes, as in numbers 1 – 4, and see if you can figure out what correcting for exponents means.

Error!

7. $4000000 \div 800 = \square$

$\square \div \square = 5 \times 10^3$

8. $270000 \div 0.09 = \square$

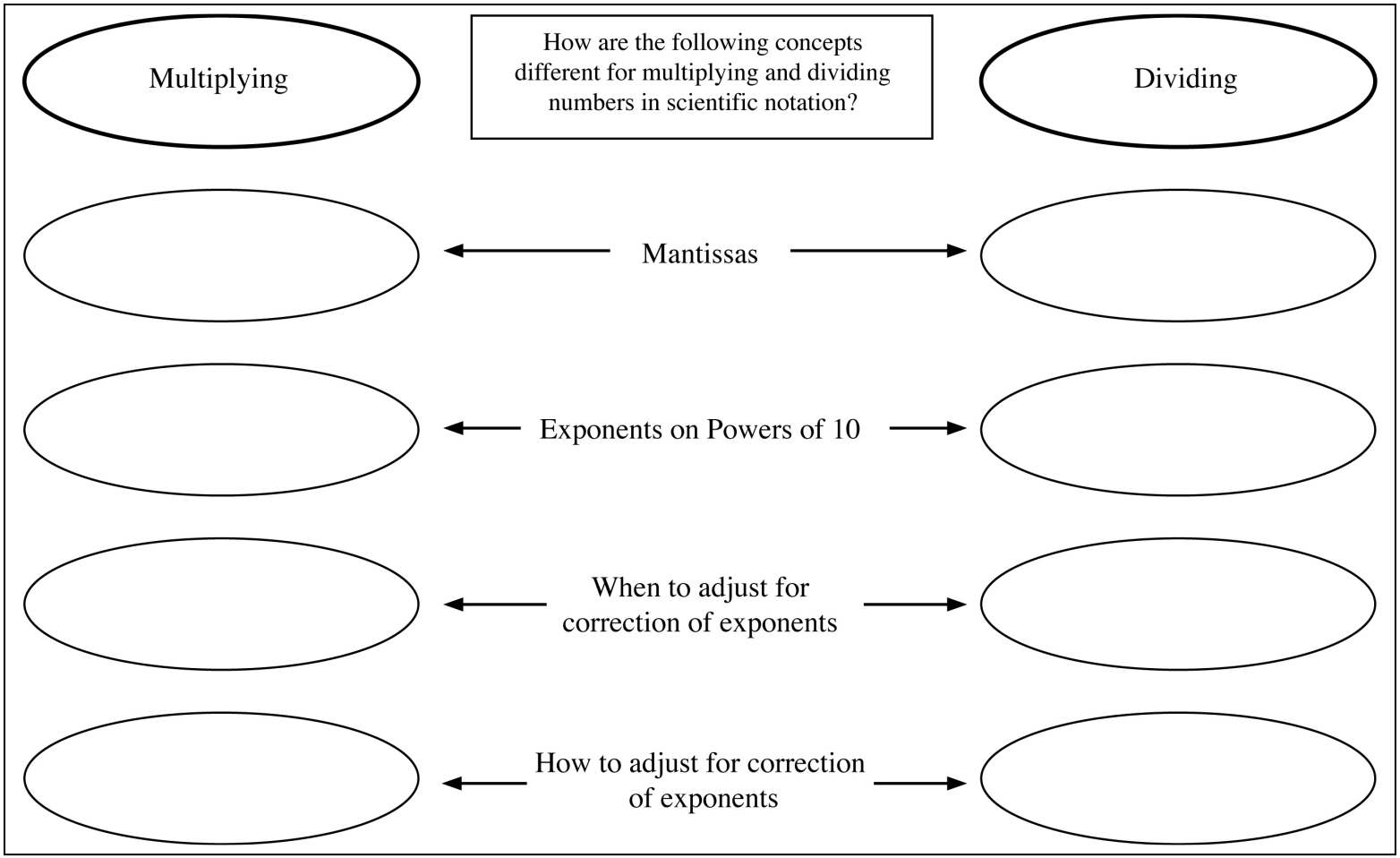
$\square \div \square = 3 \times 10^6$

9. How were questions 7 and 8 different from questions 1 -4?

10. When do you need to worry about correcting for exponents?

11. Based on your answer to question 10, create a division problem where you need to correct for exponents.

Attachment F
Multiplying and Dividing Numbers in Scientific Notation



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Attachment G
Answer Key for Attachments D, E and F

Multiplying Numbers written in Scientific Notation – Attachment D

1. 6000, 2×10^1, 3.0×10^2
2. 2500, 2.5×10^1, 1×10^2
3. .00003,1×10^{-2},3×10^{-3}
4. .00048, 2×10^{-1}, 2.4×10^{-3}
5. Multiply the mantissas and add the exponents.
6. Student answers will vary.
7. 18000; 9×10^{-1}, 2×10^2
8. 1,500,000,000,000; 5×10^6 ,3×10^5
9. The mantissa of the product is greater than 10.
10. Students will refer to what they answered in question 9.
11. Student answers will vary.

Dividing Numbers written in Scientific Notation – Attachment E

1. 16, 4×10^2, 2.5×10^1
2. 160, 3.2×10^3, 2×10^1
3. 200, 3×10^6, 1.5×10^4
4. .02, 3×10^4, 1.5×10^6
5. Divide the bases and subtract the exponents.
6. Student answers will vary.
7. 5,000; 4×10^6, 8×10^2
8. 3,000,000; 2.7×10^5; 9×10^{-2}
9. The mantissa of the quotient is less than 1.
10. Students will refer to what they answered in question 9.
11. Student answers will vary.

Graphic Organizer – Attachment F *(Sample student answers)*

Multiplication		**Division**
Multiply	**Mantissa**	Divide
Add	**Exponents**	Subtract
When multiply and get a number greater than ten.	**When to Adjust**	When divide and get number less than one.
Add one to the exponent.	**How to Adjust**	Subtract one from the exponent.

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Attachment H
The National Debt

Name:_____ **Date:**_____

Directions: Use scientific notation to solve the problems below. The bulleted facts should serve as resources to help solve the problems. Show all of the steps in your solution process.

- As of February 2004, the national debt was $7,018,431,543,709.
- The population of the United States is estimated at 293,193,870.
- Since September of 2003, the national debt has increased at an average rate of $1.88 billion dollars per day.

1. Round the national debt to the nearest trillion using whole numbers.

2. Rewrite the rounded national debt using scientific notation.

3. Round the population of the United States to the nearest ten million.

4. Rewrite the rounded population of the United States using scientific notation.

5. Rewrite the rate of increase in scientific notation.

6. What is the population of your school? Write this number using scientific notation.

7. If the national debt were to be divided equally amongst the population of the United States, how much would each person owe? (Use your rounded figures.)

8. If the national debt were to be divided equally among all the students in your school, how much would each student owe?

9. Calculate how much the national debt increases in a second.

10. If the national debt increases at a rate of $1.88 billion dollars per day, how much will it increase in one year?

11. Create your own problem involving the national debt and multiplying or dividing numbers using scientific notation.

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Attachment I
National Debt Answer Key

- As of February 2004, the national debt was recorded to be $7,018,431,543,709.
- The population of the United States is estimated at 293,193,870.
- Since September of 2003, the national debt has increased at an average rate of $1.88 billion dollars per day.

1. Round the national debt to the nearest trillion.

   $7,000,000,000,000

2. Rewrite the rounded national debt using scientific notation.

   $\$7 \times 10^{12}$

3. Round the population of the United States to the nearest ten million.

   290,000,000

4. Rewrite the rounded population of the United States using scientific notation.

   2.9×10^{8}

5. Rewrite the rate of increase in scientific notation.

   $\$1.88 \times 10^{9}$ per day

6. What is the population of your school? Write this number using scientific notation.

   Answers will vary based on school population.

7. If the national debt were to be divided equally amongst the population of the United States, how much would each person owe? (Use your rounded figures.)

$$\frac{\$7\times10^{12}}{2.9\times10^{9}} \approx \$2.41x10^{4}$$

8. If the national debt were to be divided equally among all the students in your school, how much would each student owe?

   $\$7 \times 10^{12} \div$ Population of school

9. Calculate how much the national debt increases in a second.

   1 day = 24 hours
   60 minutes =1 hour
   60 seconds = 1 minute

$$\frac{\$1.88x10^{9}}{1day}\cdot\frac{1day}{24hr}\cdot\frac{1hr}{60\,\text{min}}\cdot\frac{1\,\text{min}}{60\,\text{sec}} \approx \$21{,}759.30$$

10. If the national debt increases at a rate of $1.88 billion dollars per day, how much will it increase in one year?

$$\frac{\$1.88\times10^{9}}{1day}\cdot\frac{365days}{1year} \approx 6.86x10^{11}$$

11. Create your own problem involving the national debt and multiplying or dividing numbers using scientific notation. (Answers will vary.)

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Attachment J
Area in My Home

Name_____ **Date**_____

Directions: Gather data by measuring and record it below. Complete the questions that follow. Show all of your calculations in appropriate scientific notation.

Name of the location being measured: _____

1. Length in meters: _____ 2. Width in meters: _____

3. Area in square meters:

4. Length in centimeters:

5. Width in centimeters:

6. Area in square centimeters:

7. Length in kilometers:

8. Width in kilometers:

9. Area in square kilometers:

10. Length in miles:

11. Width in miles:

12. Area in square miles: