S. Asli Özgün-Koca, Thomas G. Edwards, and Kenneth R. Chelst

Linking LEGO and Algebra

Analyzing profit from building LEGO® pets allows students to solve an authentic problem, first concretely and then abstractly.

In mathematics, students should represent, model, and work with such real-world situations as those found in the physical world, the public policy realm, and society (CCSSI 2010). Additionally, students need to make decisions and be flexible enough to improve their decisions after analyzing realistic situations. The LEGO® Pets activity does just that.

Modeling with the Common Core's Standards for Mathematical Practice is a process that begins with identifying variables, analyzing relationships, and formulating a model, then moves into interpreting, validating, and reporting on the results (CCSSI 2010).

To bring this modeling process to life in classrooms, teachers need sources of authentic problems. But how can teachers determine the authenticity of a problem that will allow students to model and analyze a situation mathematically?

Herrington and Oliver (2000) and Lombardi (2007) have developed frameworks for authentic learning situations. They indicate that authentic problems share a number of characteristics. In particular, they—

1. have clear connections to the real world;
2. require significant investments of time and intellect;
3. use information from multiple sources and often require taking more than one perspective;
4. allow for student collaboration; and
5. require not only finding a solution but also interpreting that solution in the context of the problem.

We developed the LEGO Pets activity (see the **activity sheet** at the end of this article) in which students model a real-world problem situation, first concretely and then abstractly.

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We taught the activity in three sixth-grade classes in an affluent suburban district as well as in one sixth-grade and one seventh-grade class in a much less affluent suburban district. Approximately thirty students were in each class.

The students in all five classes were motivated, interested, and able to work through all the questions posed. Sometimes they needed no scaffolding to answer the questions; at other times, minimal scaffolding; and a few times, extensive scaffolding.

WORKING WITH LEGO BRICKS: THE CONCRETE APPROACH

When students were asked to use LEGO bricks in math class to assemble a dog and a duck, they had no problem seeing how many pieces were needed or how to build them. We started the activity by asking students what they thought the aim of assembling dogs and ducks would be. Common answers included how many animals one can build with the available pieces, how to make more animals, how to get more pieces, what to build with the available pieces, and how much profit the LEGO Pets Company can generate. At this stage, we were using these questions to help students identify the variable quantities in the problem: the number of ducks and the number of dogs to build. Later, these variable quantities were linked to algebraic variables with tabular, spreadsheet, and symbolic representations of the problem.

In the LEGO portion of the activity, we asked students to build ducks and dogs using only 6 large (2×4) and 13 small (1×2) LEGO bricks. Many were able to build 2 ducks and 1 dog using all the pieces (see **fig. 1a**). They discovered that when building 3 ducks, one small brick would not be used (see **fig. 1b**).

While students were doing LEGO

Fig. 1 Producing ducks and dogs with LEGO bricks helped motivate students to persevere through the activity.

(a)
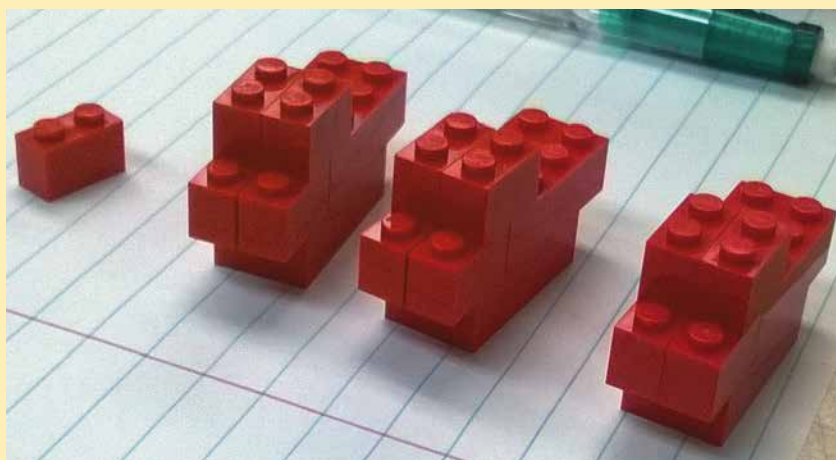
(b)

constructions, we asked them to track the combinations in a table that we provided. Our goal was to help students think about the relationships between the variables by using a table representation. The only combinations that one group found were 2 ducks and 1 dog, 3 ducks and 0 dogs, and 0 ducks and 2 dogs (see the first three rows in **fig. 2**).

When we asked about building 2 ducks and 0 dogs, one student stated that the group "would have enough pieces [remaining] to build another duck or dog, so it wouldn't be efficient." Thus, this group considered only those combinations whose left-overs would not allow them to build

Fig. 2 Combinations of animals were tracked in a table.

DUCKS	DOGS
2	1
3	0
0	2
2	0
0	1
1	1
0	0
1	0

anything else. But when we pointed out that they could include inefficient combinations, they had no problem extending the table.

We first asked students to discuss with their partner how it is possible to know, without doing any calculations, which combination would generate more profit. Later, during a whole-class discussion, many hands were raised. One student stated that the LEGO Pets Company would need to worry only about the combinations that could produce the most pets, which was 3 in this case. As a class, we decided to work with 3 ducks and 0 dogs and 2 ducks and 1 dog. Comparing these two combinations, one student explained that in both cases we could build 2 ducks, but building 1 dog would generate $3 more profit than building another duck. This discussion helped students analyze the relationship of the quantities to make a decision. After a similar discussion, each class easily determined that the maximum profit was $57, which came from building 2 ducks and 1 dog:

$$2(\$18) + 1(\$21) = \$57.$$

We next introduced the idea of *sensitivity analysis*, which is the process of analyzing a solution to see how sensitive it is to small changes in the parameters of the problem. In this situation, we asked what would happen if we had one more large or one more small piece. Many groups told us that another large piece would not help because they would need 2 additional large bricks to build either animal. Before we had a chance to ask, students told us that having one extra small piece would help; it would allow them to disassemble a duck and build a dog in its place.

The next question, which we did not include in the activity sheet but which was discussed in class, was this: "How much should you be willing to

pay for that extra small piece?" This discussion involved more analysis of the relationship among the variables, an interpretation of the results, and what decisions would improve them. Four out of five classes knew that they should pay no more than $3 for the extra small piece because replacing a duck with a dog first decreases the profit by $18, then increases it by $21.

In the other class, we needed to scaffold the discussion to help students see the no-more-than-$3 philosophy. We then introduced the term *marginal value*, which is the value of an extra unit of a resource. We also shared the fact that Leonid Kantorovich and Tjalling Koopmans, who discovered the concept of marginal value independently, won the 1975 Nobel Prize in Economics for their work (Nobelprize.org 2014).

EXPLORING WITH A SPREADSHEET: THE ABSTRACT APPROACH

We extended the activity by adding this scenario:

The LEGO Pets Company has had some success, so it decides to make more animals. To allow the company to make more animals, it has decided to have 16 large and 31 small bricks available.

First, we asked students if they would want to build the ducks and dogs by hand. The students adamantly answered "No!" This gave us a perfect opportunity to introduce the spreadsheet representation. In three classes, students had access to laptops. In the other two classes, only a demonstration laptop was available, so the spreadsheet exploration was done as a whole class. **Figure 3** displays the spreadsheet that was used in all five classes.

Students were able to make sense of the spreadsheet and connect its elements to the context of the problem. Students in all classes saw that 2 large and 4 small or 2 large and 5 small represent the number of bricks needed to make a duck and a dog, respectively. They also saw that 18 and 21 represented the profit (in dollars) on each LEGO pet and connected the term *objective function* with the objective of the problem: to generate the most profit possible. We showed them how an Excel® spreadsheet worked, and they wanted to see the formulas. When we clicked on a formula cell, all cells involved in the calculation were highlighted, which students liked.

Spreadsheet formulas can be used to introduce an algebraic representation. For example, we pointed out how the spreadsheet formula,

$$B7 * B5 + C7 * C5 = Total Profit$$

Fig. 3 A spreadsheet was also used as an organization tool.

	A	B	C	D	E	F
1	LEGO™ Toy Animals					
2	LP-max; product mix					
3		Duck	Dog			
4	Decision Variables	x1	x2			
5	Number made	0	0			
6						Total Profit
7	Objective Function	18	21			0
8						
9	CONSTRAINTS:			Pieces Used		Pieces Available
10	2x4 pieces	2	2	0	≤	16
11	1x2 pieces	4	5	0	≤	31

provides essentially the same information as the equation

$$18x + 21y = P,$$

where x represents the number of ducks made, y represents the number of dogs made, and P represents the total profit. We believe that this approach could be used in an eighth-grade class with the idea of strengthening the sophistication of the model by extending it from a table to an algebraic representation.

All classes easily found the 4 duck–3 dog combination, with $135 as the most profit. We noticed that some pairs were tracking their combinations on paper (see **fig. 4**).

When this exploration was completed as a whole-class discussion, we were able to follow students' reasoning as they moved from one combination to the next. For example, in one class, we had the following exploration:

1. 5 ducks and 3 dogs = $132 profit: We would need 4 more large bricks, so we cannot use this combination.
2. 4 ducks and 3 dogs: Yes, we can do it with a profit of $135. Two large bricks are leftover. Can we improve on it?
3. 1 duck and 7 dogs: This requires too many pieces. The spreadsheet confirmed that.

Authentic problems connect to the real world, require investments of time and intellect, use information from multiple sources, allow for student collaboration, and require both finding and interpreting a solution.

4. 1 duck and 5 dogs = $123 profit: Can we make 1 more duck? No.
5. 5 ducks and 2 dogs: We can do it, but the profit of $132 is less than $135.
6. 3 ducks and 4 dogs: We cannot build them; they require too many pieces.
7. 2 ducks and 3 dogs: $99 profit is less than $135.
8. 7 ducks and 1 dog: Too many pieces are needed.

9. Take 1 duck off, 6 ducks and 1 dog: We can do it, but the profit is only $129.
10. Build more dogs, 0 ducks and 6 dogs: We can do it, but the profit is only $126.

At this point, the students decided that 4 ducks and 3 dogs with a profit of $135 was the solution that generated the greatest profit.

When we asked students to consider the effect, if any, on the solution of having an additional large or small piece, all students in each class saw that one additional large piece would not help. Each duck and each dog required 2 large bricks. However, in one whole-class discussion, students tried 0 ducks and 7 dogs, 11 ducks and 0 dogs, and 8 ducks and 0 dogs when given one extra small piece. With the 8 ducks and 0 dogs, one extra small piece added $9 to their profit. In another class, some groups that did not find the 8 ducks and 0 dogs combination were surprised by the solution when they worked with their own laptops. Indeed, because it involved making none of the more profitable item, the solution was somewhat counter-intuitive.

Technology definitely enhanced student exploration. We believe that using the computer—even indirectly in the whole-class format—held student interest and attention. Both classroom teachers seemed pleasantly surprised by the ease with which most of the students interacted with the spreadsheet.

The technology also made it possible to introduce an algebraic approach. When we returned to the spreadsheet and interpreted cell calculations algebraically, we were able to discuss real-life connections, such as a large company perhaps having 32,000 variables and 100,000 constraints.

Fig. 4 The starred line on the combination table identified the maximum profit.

ASSESSING THE ACTIVITY'S FRAMEWORK

This activity met the criteria for an authentic task and a modeling cycle. In particular, the context of the LEGO Pets activity provided clear connections to the real world. We spent nearly an hour on one problem identifying variables and relating those variables; we then asked students to determine a solution by reasoning and decision making, not by calculating. This discussion necessitated significant investments of time and intellect.

During the activity, students did not need to use information from multiple sources, but they were required to take more than one perspective: their own and that of a business manager. The work clearly allowed students to collaborate in pairs or larger groups. The activity also required finding a solution, validating the solution, and interpreting that solution in the context of the problem. Moreover, the sensitivity analysis, asking students to consider whether one extra large or small piece would change the solution and what that value would be, goes far beyond interpreting results often found in textbook examples.

"ONE LITTLE CHANGE CAN MAKE A BIG DIFFERENCE"

Students stated that they liked using LEGOs, creating their own combinations, and using computers. They stated that the activity was fun, especially using LEGOs in math. One student wrote, "I liked this activity because we got to build out of LEGOs, actually do something instead of watching." It was interactive and helped their problem-solving skills. They learned new things, such as that "one little change can make a big difference." Many stated that the activity made them think. They suggested that we make it more challenging by adding a third animal and offered to make their own. We recommend giving students some time to do that before beginning the activity.

REFERENCES

Common Core State Standards Initiative (CCSSI). 2010. Common Core State Standards for Mathematics. Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf

Herrington, Jan, and Ron Oliver. 2000. "An Instructional Design Framework for Authentic Learning Environments." *Educational Technology Research and Development* 48 (3): 23–48. doi:http://dx.doi.org/10.1007/BF02319856

Lombardi, Marilyn M. 2007. "Authentic Learning for the 21st Century: An Overview." *Educause Learning Initiative Paper* 1:1–12.

Nobelprize.org. 2014. "The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1975." http://www.nobelprize.org/nobel_prizes/economic-sciences/laureates/1975/

Any thoughts on this article? Send an email to *mtms@nctm.org.—Ed.*

**S. Asli Özgün-Koca**, aokoca@wayne.edu, **Thomas G. Edwards**, t.g.edwards@wayne.edu, and **Kenneth R. Chelst**, kchelst@wayne.edu, are colleagues at Wayne State University in Detroit, Michigan. They are interested in providing authentic real-world problem contexts to support mathematics teaching and learning.

We want to hear from you!

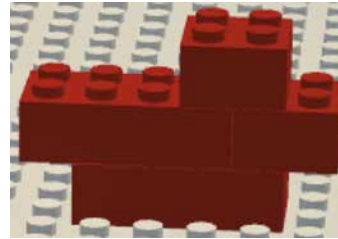Email letters to **mtms@nctm.org** and type Readers Write in the subject line.
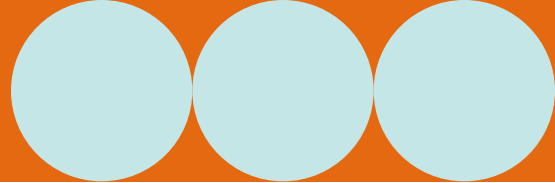
Name _____

DUCK AND DOG PRODUCTION

The LEGO Pets Company makes ducks and dogs, as shown in the figures below. The company uses 2 large (2×4) and 4 small (1×2) bricks to make 1 duck, and 2 large and 5 small bricks to make 1 dog.

Two 2×4 and four 1×2 LEGO bricks are used to make 1 duck.
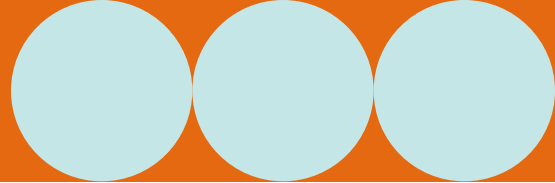
Two 2×4 and five 1×2 LEGO bricks are used to make 1 dog.
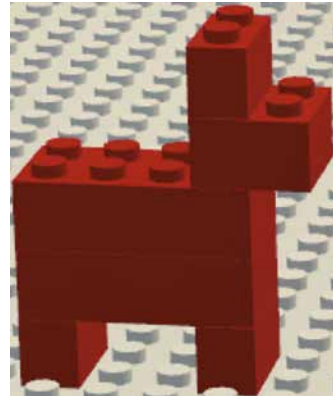
The profit on a duck is $18, and the profit on a dog is $21. LEGO Pets has 6 large and 13 small bricks available each hour. To generate the most profit, LEGO Pets must find out how many ducks and how many dogs the company should produce in one hour.

Ducks	Dogs

Name _____

1. Using 6 large and 13 small LEGO bricks, find all possible combinations of ducks and dogs that the company can make. Use the table on the previous page to record your combinations.

2. After you have figured out all possible combinations, explain how to determine which combination will generate the most profit without actually computing each combination.

3. What will be the greatest profit? _____

4. Joe thought that producing only dogs would generate the most profit. Do you agree? Explain why or why not.

5. Suppose the company could get 1 more large brick each hour. Would that allow LEGO Pets to generate more profit? Explain.

6. Suppose the company could get 1 more small brick each hour. Would that allow LEGO Pets to generate more profit? Explain.

LEGO Pets has had some success, so they decide to make more animals. To make more animals, the company has 16 large and 31 small bricks available.

7. Find how many ducks and how many dogs LEGO Pets should make to generate the most profit. How much profit is that? (Use a spreadsheet if available.)

8. Is there any value to getting 1 more large brick each hour? Explain.

9. Is there any value to getting 1 more small brick each hour? Explain.